LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **PHYSICS**

FIRST SEMESTER – **APRIL 2013**

PH 1817 - CLASSICAL MECHANICS

Date : 27/04/2013 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

PART A

Answer all questions **(10×2 = 20)**

01. What is a velocity dependent potential? Give an example.
02. Show that Newton's second law can be deduced from the Lagrange's equations.
03. The Lagrangian of a system is $L = ½mk^2\theta^2 + mgh\cos\theta$. Using the Lagrange's equations find the equation of motion for the system.
04. Show that the kinetic energy T for a torque free motion of a rigid body is a constant of motion.
05. Using the definition of $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$, show that $\mathbf{L} = \mathbf{I}\omega$
06. Show that $[u,v] = -[v,u]$
07. Show that the generating function $F_3 = pQ$ generates an identity transformation with a negative sign.
08. What is Jacobi identity?
09. Define Hamilton's principal function S
10. Using the definitions of action and angle variable show that the change in ω during a complete period is unity.

PART B

Answer any four **(4×7.5 = 30)**

11. Explain the different constraints of motion with suitable examples.
12. Reverse the Legendre's tranformation to derive the properties of L(q,q,t) from H(p,q,t) treating the q_i as independent quantities and show that it leads to the Lagrange's equation of motion.
13. Given that the generating function for a harmonic oscillator is $F_1 = (m/2)\ \omega q^2 \cot Q$. Show that the Hamiltonian of the oscillator transform to $K=\omega P$ and hence find q(t).
14. For the Kepler's problem in action-angle variables assume the expression for the action integral as $\mathbf{J}_r = [2mE + 2mk/r – (\mathbf{J}_\theta + \mathbf{J}_\varphi)^2 / 4\pi^2r^2]^{1/2}$ .dr. Solve this integral to show that $\tau^2 \propto a^3$ where τ is the time period of any planet with semi-major axis ' a ' about the Sun.
15. Write the Lagrangian for the linear triatomic molecule and solve for the normal modes of vibrations.

PART C

Answer any four **(4×12.5 = 50)**

16 a. Solve the equation of the orbit : $\theta = \ell \int dr/r^2\{2m(E- V(r) - \ell^2/2mr^2\}^{½} + \theta'$ for an attractive central potential. Classify the orbits in terms of e and E. (7.5)

b. A particle of mass m is constrained to move on the inner surface of a cone of semi-angle α under the action of gravity. Setup the Lagrangian and the equation of motion. (5)

17 a.Obtain the expression for the Coriolis effect as $2m(\boldsymbol{\omega} \times \mathbf{v_r})$ where v_r is the velocity in the rotational frame of reference. State its importance in the Earth related phenomenon. (8)

b. Prove that the moment of inertia about a given axis is related to the moment of inertia about a parallel axis passing through the centre of mass. (4.5)

18 a. Explain the theory of canonical tranformations. (4.5)

b. Show that the following tranformations are canonical.

i) $Q = p + iaq$ and $P = (p - iaq)/2ia$ ii) $2P = p^2 + q^2$ and $Q = \tan^{-1} q/p$ (8)

19 a. Solve by the Hamilton Jacobi method the motion of a particle in one dimension whose Hamiltonian is given by $H = p^2/2m + V(q)$. (7.5)

b. Give an account of fundamental Poisson's brackets. (5)

20. Write notes on any **Two** of the following

i) Euler's angles

ii) Application of the variational principle

iii) Solution to one dimensional harmonic oscillator by H-J method

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